Exercise 10

Suppose $4x^2 + 9y^2 = 36$, where x and y are functions of t.

(a) If $dy/dt = \frac{1}{3}$, find dx/dt when $x = 2$ and $y = \frac{2}{3}\sqrt{5}$.

(b) If $dx/dt = 3$, find dy/dt when $x = -2$ and $y = \frac{2}{3}\sqrt{5}$.

Solution

Differentiate both sides of the given equation with respect to t and use the chain rule.

$$\frac{d}{dt}(4x^2 + 9y^2) = \frac{d}{dt}(36)$$

$$4\frac{d}{dt}(x^2) + 9\frac{d}{dt}(y^2) = 0$$

$$4(2x) \cdot \frac{dx}{dt} + 9(2y) \cdot \frac{dy}{dt} = 0$$

$$4x\frac{dx}{dt} + 9y\frac{dy}{dt} = 0$$

Part (a)

Solve for dx/dt.

$$\frac{dx}{dt} = -\frac{9y}{4x}\frac{dy}{dt}$$

If $dy/dt = \frac{1}{3}$ and $x = 2$ and $y = \frac{2}{3}\sqrt{5}$, then

$$\frac{dx}{dt}\bigg|_{\substack{x=2 \\ y=\frac{2}{3}\sqrt{5}}} -\frac{9\left(\frac{2}{3}\sqrt{5}\right)}{4(2)}\left(\frac{1}{3}\right) = -\frac{\sqrt{5}}{4}.$$

Part (b)

Solve for dy/dt.

$$\frac{dy}{dt} = -\frac{4x}{9y}\frac{dx}{dt}$$

If $dx/dt = 3$ and $x = -2$ and $y = \frac{2}{3}\sqrt{5}$, then

$$\frac{dy}{dt}\bigg|_{\substack{x=-2 \\ y=\frac{2}{3}\sqrt{5}}} = -\frac{4(-2)}{9\left(\frac{2}{3}\sqrt{5}\right)}(3) = \frac{4}{\sqrt{5}}.$$